

VECTOR ANALYSIS

أولى
انطلا

SHEET (1)

1- Find the directional derivative of the scalar field:

$$\phi = xy^2 + yz^3$$

at the point (2, -1, 1) parallel to the vector: $\bar{i} + 2\bar{j} + 2\bar{k}$.

2- Find the unit vector normal to the surface:

$$xy^3z^2 = 4$$

at the point (-1, -1, 2).

3- Calculate the projection of the vector ∇u on the vector:

$$\frac{dx}{ds}\bar{i} + \frac{dy}{ds}\bar{j} + \frac{dz}{ds}\bar{k}.$$

4- Get $\nabla\phi$ in the following cases:

- $\phi = xyz$
- $\phi = x^2 + y^2 + z^2$
- $\phi = (x^2 + y^2 + z^2)^{\frac{1}{2}}$
- $\phi = (x^2 + y^2 + z^2)^{\frac{1}{2}}$

5- $(\nabla\phi)^2$ is defined as: $(\nabla\phi)^2 = \nabla\phi \cdot \nabla\phi$, prove that:

$$(\nabla\phi)^2 = \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2$$

6- Find the directional derivative at the point (1, 2, 3) for the scalar field:

$$\phi = x^2 + y^2 + z^2$$

parallel to the straight line: $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

7- What is the value of the maximal rate of change of the function: $u = xyz^2$ at the point (1, 0, 3).

8- Given the functions: $u = x + y + z$, $v = x + y$ and $w = -2xz - 2yz - z^2$ prove that the triple scalar product: $[\nabla u, \nabla v, \nabla w] = 0$.

9- Find the unit vector normal to the surface: $x^2y^2z^3 = 8$ at the point (1, 1, 2).

SHEET (2)

- 1- A vector field \vec{F} is defined as :

$$\vec{F}(x, y) = y \cdot \cos(2x + y^2) \vec{i} - \cos(2x + y^2) \vec{j} + x^2 y \vec{k}$$

prove that this field is solenoidal.

- 2- Find the curl of the vector field :

$$\vec{A}(x, y, z) = xy^2 \vec{i} + (x + y) \vec{j} + x^3 yz \vec{k}.$$

- 3- Prove that the vector function :

$$\vec{F}(x, y, z) = (xz^2 - xy^2) \vec{i} + (x^2 y - yz^2) \vec{j} + (y^2 z - x^2 z) \vec{k}$$

is solenoidal.

- 4- Prove that the vector field :

$$\vec{F}(x, y, z) = 2xy^2 z \vec{i} + 2x^2 yz \vec{j} + x^2 y^2 \vec{k}$$

is irrotational.

Prove also that there is a scalar potential ϕ conjugate to the field \vec{F} such that :

$$\vec{F} = -\nabla \phi$$

hence find the mathematical expression for ϕ .

- 5- Given a vector field \vec{F} defined as :

$$\vec{F}(x, y, z) = x^3 yz \vec{i} + xy^3 z \vec{j} + xyz^3 \vec{k},$$

find its curl.

Prove that this curl represents a solenoidal field.

- 6- If $\phi = xyz^2$, find its gradient, hence prove that its curl gradient vanishes

- 7- If $\phi = x^2 + y^2 - 2z^2$, prove that $\text{div grad } \phi = 0$.